Name:

Student ID:

Solution to

Quiz 3

Answer all questions. Each question carries 15 marks.

1. Let D be the parallelogram formed by the lines $x+y=-2, x+y=1, y=3x, y=3x+1$. Evaluate the line integral
$$\oint_C xy\,dx+3y\,dy$$
where C is the boundary of D oriented in anticlockwise direction. Suggestion: Try Green's theorem and then apply the change of variables formula.

By Green's thm, $\oint xy\,dx+3y\,dy = \iint_D (3y)_x-(xy)_y\,dA$
$= -\iint_D x\,dA$

Let $u=x+y$, $v=y-3x$

Then $(u,v)\mapsto(x,y)$

$R=[-2,1]\times[0,1]\mapsto D$

$\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}1&1\\-3&1\end{vmatrix}=4$

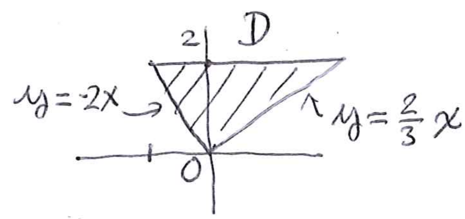
also, $x=\frac{1}{4}(u-v)$

$\therefore -\iint_D x\,dA = -\frac{1}{4}\iint_R (u-v)\times\frac{1}{4}\,dA(u,v)$

$= -\frac{1}{16}\int_{-2}^{1}\int_{0}^{1}(u-v)\,dv\,du = -\frac{1}{16}\int_{-2}^{1}\left(uv-\frac{v^2}{2}\right)\Big|_0^1 du$

$= -\frac{1}{16}\int_{-2}^{1}\left(u-\frac{1}{2}\right)du = -\frac{1}{16}\left(\frac{u^2}{2}-\frac{1}{2}u\right)\Big|_{-2}^{1} = \frac{3}{16}$ #

2. Find the outward flux of the velocity field $\mathbf{F}=xy\mathbf{i}+\sin x\mathbf{j}$ through the boundary of the triangle with vertices at $(-1,2),(3,2)$, and $(0,0)$.



Outward flux $=\oint_C \vec{F}\cdot\hat{n}\,ds$

$= \iint_D (xy)_x+(\sin x)_y\,dA$

$= \iint_D y\,dA$

$= \int_0^2\int_{-y/2}^{3y/2} y\,dx\,dy = \int_0^2 yx\Big|_{-\frac{y}{2}}^{\frac{3y}{2}}dy = 2\int_0^2 y^2\,dy$

$= 2\frac{y^3}{3}\Big|_0^2$

$= \frac{16}{3}$ #